Issues related to the computation of atmospheric angular momentum functions

Michael Schindelegger¹, Johannes Böhm¹, David Salstein², Harald Schuh¹

¹ Vienna University of Technology, Vienna, Austria
² Atmospheric and Environmental Research, Inc. Lexington, MA, U.S.A.
michael.schindelegger@tuwien.ac.at
http://ggosatm.hg.tuwien.ac.at

Introduction

Atmosphere-induced variations in Earth rotation are routinely inferred from variations of atmospheric angular momentum (AAM) functions, which can be estimated from the standard analysis fields produced by different meteorological centers. Within the project GGOS Atmosphere, which is funded by the Austrian Science Fund, AAM pressure (or matter) terms were determined from the analysis data of the European Centre for Medium-Range Weather Forecasts (ECMWF). For the calculation of these matter terms, both a three-dimensional integration including pressure increments in the vertical as well as the established two-dimensional integration over just surface pressure values were applied. Time varying differences exist between the matter terms from both methods, implying a significant effect on Earth rotation parameters – 1 mas in case of polar motion and about 3 µs for changes of LOD. The observed AAM anomalies originate from the approximative nature of the simplified, two-dimensional integration, which ignores the variability of the mass distribution with height, giving a slightly wrong estimate of the atmospheric moment of inertia. It is shown, that this deficiency can be corrected if the surface pressure is evaluated at the center of mass (COM) of each atmospheric column. ‘Mapping’ the surface pressure onto the COM gives a correct two-dimensional integration method for AAM pressure terms.

Calculation methods

Determining the matter terms of AAM functions demands the evaluation of the moment of inertia Ic over radius increments dr, stretching from the Earth’s surface r, up to ∞ (see Eq.1). Assuming hydrostatic equilibrium, Ic can be likewise derived from integration over pressure increments dp (Eq.2). If radius r and gravity g are replaced by their respective constant values at the surface (subscript 0), the vertical integral is equivalent to the surface pressure ps of the column (Eq.3). Abbreviations for each calculation method are given below:

dr:                                   (1)
dc: = ∫ρgdr                                  (2)
ps: = r²0g                                  (3)

Note: Summing up the column-wise moments of inertia over the sphere yields the two- and three-dimensional integration mentioned in the introduction.

Graphical comparison

For each of the data sets, dimensionless AAM matter functions X0, X1, X2 were obtained – both from three-dimensional integration over pressure increments dp (following the moment of inertia in Eq.2) and from the surface pressure integral ps (Eq.3). Figure 1 and 2 depict the difference of these two methods on the level of AAM functions.

Analytical comparison

A reasoning for the discrepancies can be given straightforward: for the atmospheric moment of inertia (Eq.1-3), the vertical distribution of air masses is pivotal, but only with dr and dp this is considered properly. The surface pressure method instead, compresses the atmospheric mass onto the Earth’s surface and thus provides only an approximation of the real value of the moment of inertia. The approximation error can be quantified, if in Eq.3, the vertical integral of density over height dh is replaced by the vertical integral of density over height increments dh (Eq.5) and similarly, Eq.2 is rewritten in terms of dh (Eq.4). Expanding in Eq.4 yields five terms, of which only the first one is reproduced in the ps-method. The second term has a considerable size, too – 0.5% of the first one – enough to be responsible for the discrepancies between dp and ps.

dp:                                   (4)
ps:                                   (5)

Conclusions

The deviations between the rigorous integration and the surface pressure method amount to 0.5% of the plain signal in the matter terms of AAM, or equivalently, 1 mas on the level of polar motion and 2.6 µs on the level of LOD.

Shifting the surface pressure to the COM gives a correct moment of inertia and consequently also the same AAM as obtained by strict vertical integration.

For the project GGOS Atmosphere, a global 1° by 1° grid containing the height of the COM will be provided on an operational basis, four times a day. It is thought to apply these data sets also for the calculation of atmospheric gravity corrections.

Reference:

Acknowledgement:
GGOS Atmosphere (P20902) is funded by the Austrian Science Fund (FWF).