Impact of A Priori Gradients on VLBI-Derived Terrestrial Reference Frames

J. Böhm, H. Spicakova, L. Urquhart, P. Steigenberger, H. Schuh

Abstract We compare the influence of two different a priori gradient models on the terrestrial reference frame (TRF) as determined from Very Long Baseline Interferometry (VLBI) observations. One model has been determined by vertical integration over horizontal gradients of refractivity as derived from data of the Goddard Data Assimilation Office (DAO), whereas the second model (APG) has been determined by ray-tracing through monthly mean pressure level re-analysis data of the European Centre for Medium-Range Weather Forecasts. We compare VLBI solutions from 1990.0 to 2011.0 with fixed DAO and APG gradients to a solution with gradients being estimated, and we find better agreement of station coordinates when fixing DAO gradients compared to fixing APG gradients. As a consequence, we recommend that gradients are constrained to DAO gradients, in particular in the early years of VLBI observations (up to about 1990), when the number of stations per session is small and the sky distribution is far from uniform. Later than 1990, the gradients can be constrained loosely and the a priori model is of minor importance.

Keywords Troposphere Gradients, Terrestrial Reference Frame, VLBI

1 Introduction

As recommended by the Conventions of the International Earth Rotation and Reference Systems Service (IERS) (Petit and Luzum, 2010), the line-of-sight delay, D_L, is expressed as a function of four parameters as follows:

$$\begin{aligned} D_L = {} & m_h(e)D_{hz} + m_w(e)D_{wz} \\ & + m_g(e)[G_N\cos(a) + G_E\sin(a)]. \end{aligned} \quad (1)$$

The four parameters in this expression are the zenith hydrostatic delay, D_{hz}, the zenith wet delay, D_{wz}, and a horizontal delay gradient with components G_N (north) and G_E (east). m_h, m_w, and m_g are the hydrostatic, wet, and gradient mapping functions, respectively, and e is the elevation angle of the observation direction in vacuum. a is the azimuth angle in which the signal is received, measured clockwise from north.

Horizontal gradient parameters are needed to account for the systematic component in the north-south direction towards the equator due to the atmospheric bulge (MacMillan and Ma, 1997), and they also capture the effects of random components in both directions due to the variable weather systems. Usually, those gradients are estimated in the analysis of Global Navigation Satellite Systems (GNSS) and Very Long Baseline Interferometry (VLBI) observations with the gradient mapping function m_g as part of the partial derivative.

Although it is generally not necessary to constrain those gradient estimates to a priori values in GNSS analysis, it is recommended to constrain the estimates in the early years of VLBI observations up to about 1990 (Spicakova et al., 2011) when only a few stations were observing per session and the distribution of the observations in the sky per station was far from uniform. In any case, it is advisable to constrain the gradient estimates to a realistic a priori gradient model which accounts for the atmospheric bulge. In the next sections, we describe the effect of constraining (fixing) the gradient estimates to a priori models different from zero.

2 Gradient mapping function

Two types of gradient mapping functions m_g are widely used in GNSS and VLBI software packages. On the one hand, there is the formulation by MacMillan (1995) which goes back to Davis et al. (1993) for the ”wet” refractivity of air:

$$m_g = \cot(e)m_h. \quad (2)$$

Johannes Böhm, Hana Spicakova, and Harald Schuh
Vienna University of Technology, Gußhausstraße 27-29, A-1040 Vienna, Austria, johannes.boehm@tuwien.ac.at
Landon Urquhart
University of Calgary, Canada
Peter Steigenberger
Technische Universität München, Germany

This approach has the disadvantage that it is singular at the horizon. On the other hand, Chen and Herring (1997) suggest applying

$$m_g = \frac{1}{\tan(e)\sin(e) + C} \quad (3)$$

and they recommend to use $C = 0.0032$ for the estimation of total gradients (Herring, 1992). This formulation is based on a theoretical concept with an exponential decay of the horizontal gradient of refractivity with height. If used with the coefficient $C = 0.0007$ (corresponding to a scale height of 3 km) it describes the gradient mapping function for the wet part and is rather close to the formulation by MacMillan (1995) (see also Figures (1) and (2)).

Exemplarily, we determined ray-traced delays at 5, 7, 10, 15, 20, 30, 50, 70, and 90 degrees elevation towards north, east, south, and west at two sites (Wettzell in Germany, and Tsukuba in Japan) on 1 January 2008 at 0 UT. Then, we removed the azimuth-symmetric part and compared the residuals to the gradient mapping functions by scaling the latter so that they agree with the residual ray-traced delays at 5 degrees elevation. From the two samples we did not find a clear preference of one type of gradient mapping function (see Figures (1) and (2)). Furthermore (not shown here), the impact on station coordinates is at the sub-millimetre level if either using the formulation by MacMillan (1995) (Eq. (2)) or Chen and Herring (1997) (Eq. (3)) for the estimation of gradients. Thus, we used the formulation by Chen and Herring (1997) (Eq. (3)) with $C = 0.0032$ for our investigations, i.e., for mapping the a priori gradients as well as for estimating gradients, and we recommend its application in all software packages for better comparability.

3 A priori gradient models

We used two different a priori gradient models for comparison. MacMillan and Ma (1997) introduced gradients derived from data of the Goddard Data Assimilation Office (DAO) (Schubert et al., 1993). These gradients are derived by vertical integration over the horizontal gradients of refractivity, and they are provided for all VLBI sites. Secondly, Böhm et al. (2011b) determined an a priori gradient (APG) model from 40 Years Re-Analysis (ERA40) monthly mean pressure level data of the European Centre for Medium-Range Weather Forecasts (ECMWF) with a horizontal resolution of 5 degrees. North and east gradients were derived by the determination of ray-traced delays in zenith direction as well as towards north, east, south, and west at 5 degrees elevation, and by fitting those delays to the model by Chen and Herring (1997) (Eq. (3)) with the coefficient $C = 0.0032$. The north and east gradients, G_N and G_E, were then averaged over all months and expanded into spherical harmonics up to degree and order 9. This model can be downloaded from the GGOS Atmosphere Server at TU Vienna [1] and plots on APG are provided by Böhm et al. (2011b).

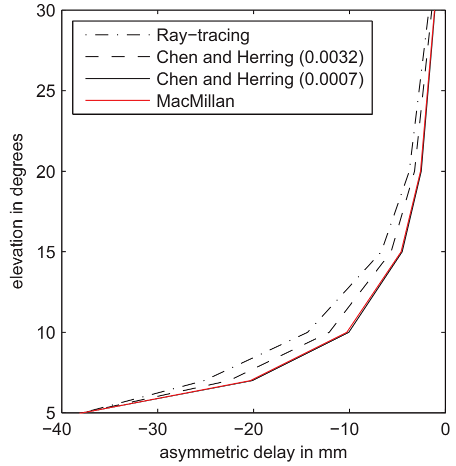

Fig. 1 Asymmetric delays towards east at station Wettzell on 1 January 2008. The gradient mapping functions were scaled to agree with the ray-traced delays at 5 degrees elevation. A coefficient C of about 0.0060 would agree best with the ray-traced delays in the range of elevations shown in the figure.

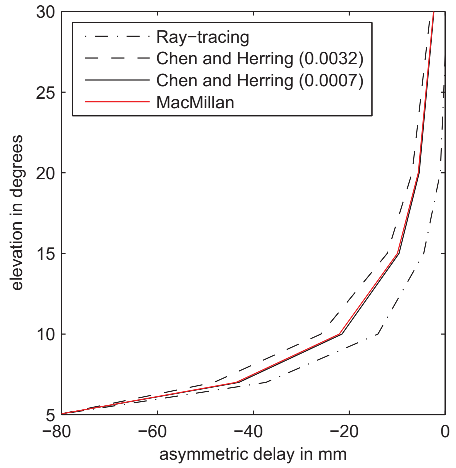

Fig. 2 Asymmetric delays towards west at station Tsukuba on 1 January 2008. The gradient mapping functions were scaled to agree with the ray-traced delays at 5 degrees elevation. A coefficient C of about -0.0030 would agree best with the ray-traced delays in the range of elevations shown in the figure.

Figure (3) shows north gradients from DAO (MacMillan and Ma, 1997) and APG (Böhm et al., 2011b) for all VLBI stations. Clearly visible is the atmospheric bulge above the equator which causes the north gradients to be slightly negative in the northern and slightly positive in the southern hemisphere. However, there

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[1] http://ggosatm.hg.tuwien.ac.at/DELAY/SOURCE/apg.f

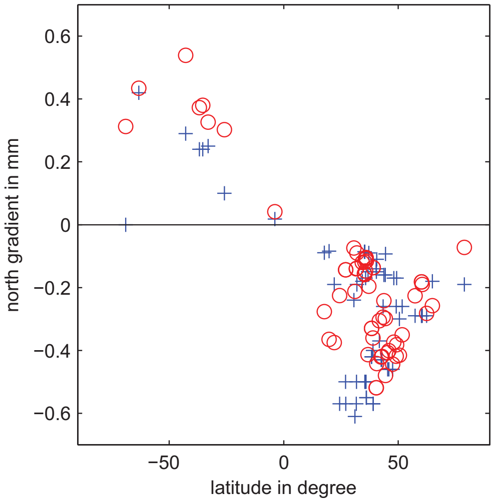

Fig. 3 North gradients in mm versus station latitude as derived from DAO by vertical integration (+) and by ray-tracing through ECMWF data as provided with APG (o).

is a quite good agreement between the gradients derived by two completely different approaches and different weather models.

4 VLBI analysis

Böhm et al. (2011b) carried out investigations with APG in the analysis of Global Positioning System (GPS) observations, and they found that the north gradients from APG are generally larger than the north gradients estimated in GPS analysis. The reason for this is not clear, but possible contributions might come from the sky distribution at the stations which is not uniform or from the downweighting of observations at low elevations (Urquhart et al., 2011). Spicakova et al. (2011) showed the importance of constraints (on zero a priori gradients) in the early years of VLBI observations up to about 1990. If those constraints are not applied, estimated gradients get unrealistically large and impact station coordinates significantly.

We compared three different VLBI solutions for the years 1990.0 to 2011.0 (Spicakova et al., 2011) obtained with the Vienna VLBI Software (VieVS) (Böhm et al., 2011a). In the first solution we estimated gradients as piecewise linear offsets every six hours with relative constraints (0.5 mm after six hours) but without absolute constraints. In the second solution we fixed the gradients to the values from APG, and in the third solution we fixed the gradients to the values from DAO. The criterion for a good a priori gradient model is that the estimated station coordinates are close to those station coordinates estimated in the first solution.

Figure (4) shows the differences in the north components with respect to the first solution. Clearly visible are the smaller differences in the station north components for DAO gradients compared to APG gradients, in particular in Asia and Europe. This is also confirmed by the station up components shown in Figure (5).

However, as soon as gradients are estimated (with no or loose constraints), it is no longer of importance which a priori gradients are used and the station coordinates agree.

5 Conclusions

We recommend using the gradient mapping function as introduced by Chen and Herring (1997) (Eq. (3)) with the coefficient $C = 0.0032$ for the mapping of a priori gradients as well as for the estimation of gradients. The application of the same gradient mapping function by different space geodetic techniques is the prerequisite for a rigorous combination of gradients.

For the analysis of VLBI sessions up to 1990, we recommend constraining the gradient estimates to DAO gradients, as introduced by MacMillan and Ma (1997). After 1990, when the number of stations per session is larger and the sky distribution with sources at the stations is more uniform, the choice of the a priori gradients is less important because gradients can be estimated reliably.

There is the plan to revise the gradient model APG by using a better resolution than just degree and order 9, or - similar to DAO - to determine those gradients for every station specifically. On the other hand, we will investigate and revive the application of the six-hourly linear horizontal gradients (Böhm and Schuh, 2007) which are available for all VLBI sites since 2006 at the GGOS Atmosphere Server.

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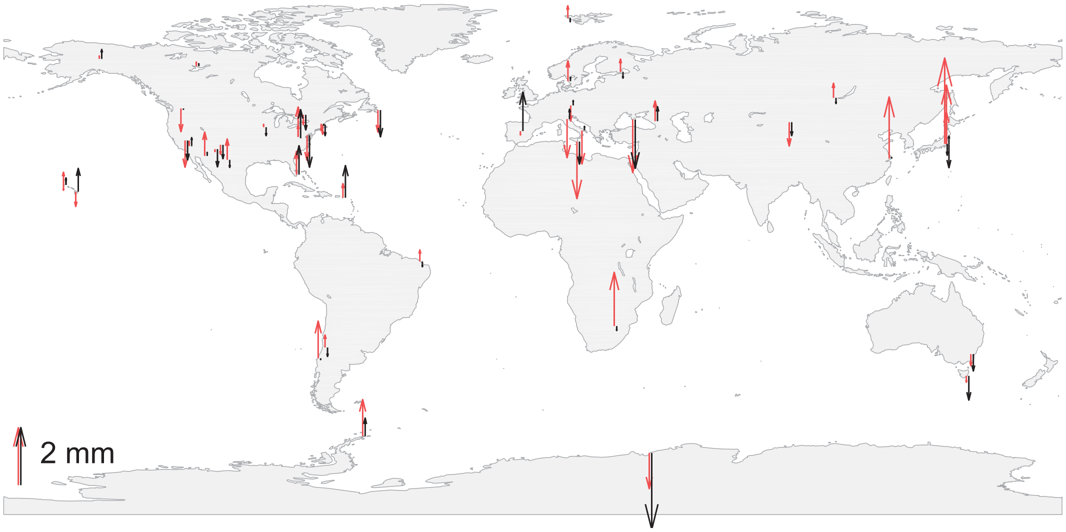

Fig. 4 Station north components with fixed APG (red) and DAO gradients (black), compared to a solution with gradients estimated.

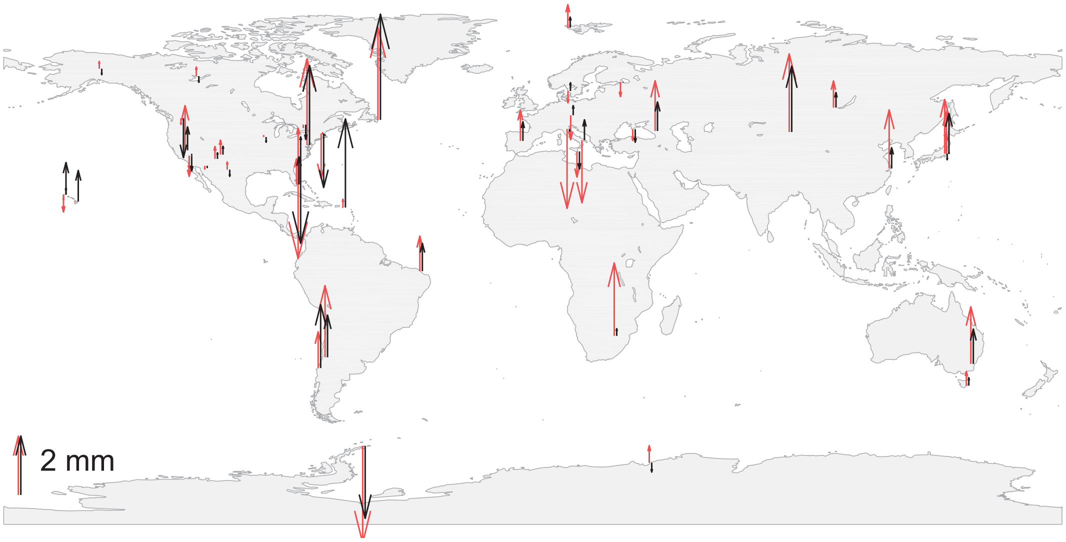

Fig. 5 Station up components with fixed APG (red) and DAO gradients (black), compared to a solution with gradients estimated.

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